

Code No: 113AH

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November - 2015

MATHEMATICS-III

(Common to EEE, ECE, EIE, AGE, ETM)

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART- A**(25 Marks)**

- 1.a) Determine the nature of the point $x=0$ for the equation $x^2(x^2+1)y''+(x^2-1)y'+2y=0$. [2M]
- b) Find the series solution of the equation $y''-y=0$. [3M]
- c) Express $2-3x+4x^2$ in terms of Legendre polynomial. [2M]
- d) Express $J_3(x)$ in terms of J_0 and J_1 . [3M]
- e) Prove that $f(z)=\bar{z}$ is not analytic at any point. [2M]
- f) Show that the function $f(z)=\sin x \cosh y + i \cos x \sinh y$ is continuous as well as analytic every where. [3M]
- g) State the Cauchy's Residue theorem. [2M]
- h) Expand $\log z$ by Taylor's series about $z=1$. [3M]
- i) Define conformal transformation. [2M]
- j) Find the points at which $w=\cosh z$ is not conformal. [3M]

PART-B**(50 Marks)**

- 2.a) Obtain the series solution of the equation $y''+xy'+y=0$
- b) Find the series solution of $4xy''+2y'+y=0$. [4+6]

OR

- 3.a) Solve in series the equation $y''+xy=0$
- b) Solve in series the equation $(1-x^2)-2xy+n(n+1)y=0$ about $x=0$. [4+6]

- 4.a) Prove that $nP_n=(2n-1)xP_{n-1}-(n-1)P_{n-2}$, $n \geq 2$
- b) State and prove generating function of Bessel's function. [5+5]

OR

- 5.a) Prove that $\int_{-1}^1 P_n(x)P_m(x)dx=0$, if $m \neq n$, $2/(2n+1)$ if $m=n$.
- b) Prove that $J_0^2+2(J_1^2+J_2^2+J_3^2+\dots)=1$. [5+5]

- 6.a) Find the analytic function whose real part is $u=e^{2x}(x \cos 2y - y \sin 2y)$.
- b) Evaluate $\int_C \operatorname{Re} z \, dz$ where C is the shortest path from $1+i$ to $3+2i$. [5+5]

OR

- 7.a) State and prove Cauchy's integral theorem.
- b) Evaluate using Cauchy's integral formula $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$, where C is the circle $|z|=3$. [5+5]

- 8.a) Express $f(z) = \frac{z}{(z-1)(z-3)}$ in series of positive and negative powers of $(z-1)$.
- b) Evaluate $\int_0^{2\pi} \frac{1}{(5-3\cos\theta)} d\theta$ using residue theorem. [5+5]

OR

- 9.a) Give two Laurent's series expansions in powers of z for $f(z) = \frac{1}{(1-z)z^2}$ and specify the region in which these expansions are valid.
- b) Evaluate $\int_C \frac{z^2 + 2z - 2}{z(z-4)(z-1)} dz$ where C is $|z| = 1.5$. [5+5]
- 10.a) Under the transformation $w = 1/z$ find the image of the circle $|z - 2i| = 2$.
- b) Find the bilinear transformation which maps the points $(-1, 0, 1)$ into the points $(0, i, 3i)$ [5+5]

OR

- 11.a) Find the image of the region in the z -plane between the line $y = 0$ and $y = \pi/2$ Under the transformation $w = e^z$.
- b) Show the bilinear transformation preserves the cross ratio. [5+5]

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