

Code No: 123AA

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November/December - 2016

MATHEMATICS-II

(Common to CE, MMT, AE, PTE, CEE)

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

- 1.a) What is the greatest rate of increase of $\phi = xyz^2z^2$ at the point $(-1,1,2)$? [2]
- b) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $r = |\vec{r}|$. [3]
- c) Write the Euler's formula in the interval $(c, c+2\pi)$, for finding Fourier series. [2]
- d) Find the value of a_0 for the function $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$. [3]
- e) Evaluate $\frac{d^4}{dx^4} e^x$. [2]
- f) Express the function $f(x) = 2x^4 - 6x^3 + 5x^2 - 20x + 10$ in factorial notation. [3]
- g) Show that the rate of convergence of Bisection method is linear. [2]
- h) Establish Newton Raphson's method for determining the approximate value of the root of the equation $f(x) = 0$. [3]
- i) Write Simpson's $\frac{1}{3}$ rule. [2]
- j) Evaluate K_3 for the equation $\frac{dy}{dx} = y - x$, $y(0) = 1.5$ by using Runge-Kutta 4th order method. [3]

PART-B

(50 Marks)

- 2.a) Find the directional derivative of $f = xy + yz + zx$ in the direction of vector $i + 2j + 2k$ at the point $(1,2,0)$.
 - b) Find the scalar potential of $\vec{F} = (z + \sin y)\vec{i} + (-z + x \cos y)\vec{j} + (x - y)\vec{k}$. [5+5]
- OR**
- 3.a) Prove that $(y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is both solenoidal and irrotational.
 - b) Find the flux of the vector field $\vec{A} = (X - 2Z)\vec{i} + (x + 3y + z)\vec{j} + (5x + y)\vec{k}$ through the upper side of the triangular ABC with vertices at the points $A(1,0,0)$, $B(0,1,0)$, $C(0,0,1)$ [5+5]

4.a) Obtain a Fourier expansion for $\sqrt{1 - \cos x}$ in $-\pi < x < \pi$.

b) Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| \geq a \end{cases}$ where a is a positive real

number. Hence deduce that: i) $\int_0^\pi \frac{\sin t}{t} dt = \frac{\pi}{2}$ and ii) $\int_0^\pi \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$. [5+5]

5.a) Express $\cos x$ in Fourier series in $0 < x < 2\pi$.

b) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} x^2 & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases}$. [5+5]

6.a) Find the cubic polynomial interpolation which takes on the values: $f_0=5, f_1=1, f_2=9, f_3=25, f_4=55$.

b) The mode of a certain frequency curve $y = f(x)$ is very near $x = 9$ and the value of the frequency density $f(x)$ for $x=8.9, 9.0$ and 9.3 are respectively equal to $0.30, 0.35$ and 0.25 . Calculate the approximate value of the mode. [5+5]

OR

7.a) From the following table, find the number of students who obtained less than 45 marks:

Marks	30-40	40-50	50-60	60-70	70-80
No of Students	31	42	51	35	31

b) Fit a second degree parabola to the following data, taking x as the independent variable. [5+5]

x :	2	3	4	5	6	7	8	9	
y :	2	6	7	8	10	11	11	10	9

8.a) Evaluate $\sqrt{29}$ by Newton-Raphson formula. Correct to four places of decimals.

b) Apply Gauss-Seidal iteration method to solve equations.

$10x_1 + x_2 + x_3 = 12, 2x_1 + 10x_2 + x_3 = 13$ and $2x_1 + 2x_2 + 10x_3 = 14$. [5+5]

OR

9.a) By iteration method, find the root of $\tan x = x$ up to four decimal places.

b) Apply Jacobi iteration method to solve equations.

$27x + 6y - z = 85, 6x + 15y + 2z = 72$ and $x + y + 54z = 110$. [5+5]

10.a) Calculate the approximate value of $\int_0^{\frac{1}{2}\pi} \sin x \cdot x dx$.

i) By Trapezoidal rule

ii) By Simpson's rule, Using 11 ordinates.

b) Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ with the initial condition $y=0$ when $x=0$, use Picard's method to obtain y for $x=0.25, 0.5$ and 1.0 correct to three decimal places. [5+5]

OR

11.a) Use Simpson's three-eights rule to obtain the value of $\int_0^{0.3} (1 - 8x^3)^{\frac{1}{2}} dx$.

b) Solve the boundary-value problem $y'' = y(x), y(0) = y(1) = 0$ by the shooting method. [5+5]