

## B.Tech I Year (R13) Supplementary Examinations June 2017

**MATHEMATICS - I**

(Common to all branches)

Time: 3 hours

Max. Marks: 70

**PART – A**  
(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

- Solve  $(1 - x^2) \frac{dy}{dx} - xy = 1$ .
- Solve  $(xy^2 - e^{1/x^2}) dx - x^2 y dy = 0$ .
- Show that the radius of curvature at any point of the cycloid  $x = a(\theta + \sin \theta)$ ,  $y = a(1 - \cos \theta)$  is  $4a \cos \theta/2$ .
- Find the maximum and minimum values of  $3x^4 - 2x^3 - 6x^2 + 6x + 1$  in the interval  $(0, 2)$ .
- Evaluate  $\iint_A xy \, dx \, dy$ , where A is the domain bounded by x-axis, ordinate  $x = za$  and the curve  $x^2 = 4ay$ .
- Find by triple integration, the volume of the sphere  $x^2 + y^2 + z^2 = a^2$ .
- Find the Laplace transform of the function  

$$f(t) = \sin \omega t, 0 < t < \pi/\omega$$

$$= 0, \quad \frac{\pi}{\omega} < t < 2\pi/\omega$$
- Evaluate  $L \left\{ e^{-t} \int_0^t \frac{\sin \omega t}{t} dt \right\}$ .
- If  $u = x + y + z$ ,  $C = x^2 + y^2 + z^2$ ,  $w = yz + zx + xy$ . Prove that grad u, grad v and grad w are coplanar.
- Prove that  $\text{div}(r^n R) = (n + 3)r^n$ . Hence show that  $R/r^3$  is solenoidal.

**PART – B**

(Answer all five units, 5 X 10 = 50 Marks)

**UNIT – I**

- Solve  $\frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = (1 - e^x)^2$ .
  - A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original?

**OR**

- Solve by the method of variation of parameters,  $\frac{d^2 y}{dx^2} - y = \frac{2}{(1+e^x)}$ .
  - An uncharged condenser of capacity C is charged by applying an e.m.f.  $E \sin t / \sqrt{LC}$ , through leads of self-inductance L and negligible resistance. Prove that at any time t, the charge on one of the plates is  $\frac{EC}{2} \left\{ \sin \frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos \frac{t}{\sqrt{LC}} \right\}$ .

**UNIT – II**

- Using Maclaurin series, expand  $\tan x$  up to the term containing  $x^5$ .
  - Find the radius of curvature at the point  $(3a/2, 3a/2)$  of the Folium  $x^3 + y^3 = 3axy$ .

**OR**

- Find the volume of the largest possible right-circular cylinder that can be inscribed in sphere of radius a.
  - If  $y_1 = \frac{x_2 x_3}{x_1}$ ,  $y_2 = \frac{x_3 x_1}{x_2}$ ,  $y_3 = \frac{x_1 x_2}{x_3}$ , show that the Jacobian of  $y_1, y_2, y_3$  with respect to  $x_1, x_2, x_3$  is 4.

**UNIT – III**

- Trace the curve  $y = x^3 - 12x - 16$ .
  - By changing the orders of integration of  $\int_0^\infty \int_0^\infty e^{-xy} \sin px \, dx \, dy$  show that  $\int_0^\infty \frac{\sin px}{x} dx = \frac{\pi}{2}$ .

**OR**

- Find the volume of the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .
  - Find the value of the portion of the sphere  $x^2 + y^2 + z^2 = a^2$  using inside the cylinder  $x^2 + y^2 = ay$ .

## UNIT – IV

- 8 (a) If  $f(t)$  is a periodic function with period  $T$ , then prove that  $L(f(t)) = \int_0^T \frac{e^{-st} f(t) dt}{1 - e^{-sT}}$ .
- (b) Find the inverse Laplace transform of  $\frac{5s+3}{(s-1)(s^2+2s+5)}$
- OR**
- 9 (a) Using convolution theorem, evaluate  $L^{-1}\left\{\frac{1}{(s-2)(s+2)^2}\right\}$ .
- (b) Solve  $\frac{d^2x}{dt^2} + 9x = \cos 2t$ , if  $x(0) = 1$ ,  $x\left(\frac{\pi}{2}\right) = -1$

## UNIT – V

- 10 (a) Show that  $r^\alpha R$  is any irrotational vector for any value of  $\alpha$  but is solenoidal if  $\alpha + 3 = 0$ , where  $R = xi + yj + 2k$  and  $r$  is the magnitude of  $R$ .
- (b) Show that  $\nabla^2(r^n) = n(n+1)r^{n-2}$ .
- OR**
- 11 (a) Evaluate  $\int_S F \cdot N ds$ , where  $F = 2x^2yi - y^2j + 4xz^2k$  and  $S$  is the closed surface of the region in the first octant bounded by the cylinder  $y^2+z^2=9$  and the planes  $x=0$ ,  $x=2$ ,  $y=0$  and  $z=0$ .
- (b) Verify divergence theorem for  $F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$  taken over the rectangular parallelepiped,  $0 \leq x \leq a$ ,  $0 \leq y \leq b$ ,  $0 \leq z \leq c$ .

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